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GTO, the Value of Information, and the Nature of the Solution to No-limit Hold 'em

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Is GTO (game theory optimal) a way of life? Solvers and computational robots, aka BOTs, are now a staple of the online poker ecosystem. Even in live play, I often find myself in conversations with poker players aspiring to discuss poker concepts rationally. Poker has adopted game theory concepts and terminology, with phrases like Game Theory Optimal evolving to represent other related concepts – that is understandable and how language works. Nonetheless, with the use and abuse of "solvers", PioSolver a popular example, there is widespread misunderstanding. I personally think solvers have led to net poorer play at all but the highest levels. Even there, the counterfactual experiment would be required to prove they have made a difference -- in their absence poker theory would have evolved thoughtfully and rationally albeit with different data.

First, I will clarify what is understood about poker theory broadly in the context of no-limit Texas hold 'em (NLH). Heads-up play is now played most proficiently by computational algorithms -- by a BOT . The play is complex, including offering insights that have yet to be rationalized. This form of poker has a GTO solution and a formal Nash equilibrium. Still, the BOTs make approximations and are explicitly built to handle only certain bet sizing and have to estimate the value of a move that is out of their training set. Heads-up limit hold 'em is a numerically exactly solved subset of NLH. Extant NLH BOTs also often employ other game abstractions but are nonetheless better than humans in almost all cases. Even in multiplayer NLH, Bots are emerging as dominant with the results published in the world's best scientific journals.

A simple yet formidable BOT available for public play and training is PokerSnowie, a poker artificial intelligence that works with severe restrictions yet is still unbeatable for almost anyone. This BOT is evolved, via a neural network, to employ only a subset of wager sizes: checking, ¼, ½, 1x, 2x pot (or all-in if less than 2x pot), for each betting action. It is trained vs. a myriad of strategies but limits the space of its response to those bets. It is further restricted to only employing a single bet size for all of its holdings at a particular game state. Game state is defined as the cards of each player, their chip stacks and the community cards with the action at a defined location. This means that this BOT is restricted to selecting one of its bet sizes for the entire range of hands it might hold at a particular node of the game tree. To be clear, it has a checking and betting range on postflop streets and uses only one of its available sizings for every betting hand. When the expected value (EV) of checking and betting is equal, it will randomize that action but still use only a single bet sizing when aggressing. This remains true even when another bet size has the same or higher EV for a particular card combination. The EV of its betting range is being optimized, not that of the particular holding.

This type of approximation is often used commonly by human players preflop where one may open the betting in a hand using a consistent bet size. One might bet three big blinds from a particular position with all the card combinations they chose to continue with and fold the rest. PokerSnowie never splits its ranges and works under this constraint at all streets – it optimizes for all the holdings that will bet at each node of the game tree. We do not want to emulate its behavior in this regard; who wants to use the same bet size for top pair and a full house on the same board? Nonetheless it makes clear how card combinations in a range each work together to support an action at each game state. This paradigm is formidable to face even in this simple realization.

Consider, selecting a uniform bet size preflop prevents opponents from gaining insights into what hands we have when opening the betting – they only recognize that we chose to bet with a group of holdings. While an opponent can safely assume a reasonable strategy has AA in its UTG opening range and not 27, a three big blind UTG open may be AA, 77 or T9s. The opponent can't simply attack the weaker holdings or fold to the AA. Further, while AA can support a larger opening size with a higher profit, using a particularly large size for just AA would allow opponents to counter-strategize. Especially when stacks are deep, knowing the exact holding of our opponents would allow effective counter strategies and well-designed exploits. That is not to say that the pre-flop opener could not optimally defend a range of only AA but this approach would necessarily weaken the preflop range now depleted of its strongest holding. In practice, these situations arise in far from ideal play where opponents might open larger with strong holdings to limit the number of callers and "protect" their strong preflop hands.

Let me define some terms formally. An equilibrium situation in poker is one with game states that are, for example, iterated to consistency such that all the strategies become invariant in the self-consistent process. They can do no better or worse vs. each other. There will be an EV associated with each strategy at equilibrium. An optimal equilibrium evolves the strategy such that the EV is a maximum under the constraints (e.g. bet sizes, stacks, ranges) input into the solving. One might optimize one or all of the strategies and associated EV. If all the strategies are simultaneously optimized, one gets an optimal equilibrium that, in a zero-sum symmetric game, has to yield zero EV for all the strategies when a stable solution is present. This is known to be true in heads-up NLH where there is a GTO solution and Nash equilibrium, and is more complicated in multi-player versions. Still, such equilibria are found in computational simulations of multiplayer NLH and the situation has been considered formally. Any play outside these strategies is a non-equilibrium excursion, even if the play is part of a different game state's equilibrium – that is how equilibrium works. Fluctuations from equilibrium are represented at some frequency in other equilibria. In non-equilibrium dynamics, many things are possible – see below. All this is to say, any reference to this body of knowledge is summed up in most poker conversation colloquially by calling anything vaguely equilibrium-like GTO. Reality is richer and more complex. Even the zero EV set of strategies will not be the ultimate solution if constraints on the solution space are present like limiting the choice of bet sizes.

There is a strong rationale for betting in ranges where hands can be lumped together supporting some optimal bet sizing. They offer both information hiding and strategic flexibility. The combinations can be considered to support each other. In our preflop opening range, AA will robustly hold a lot of equity on many flops, e.g. vs. a single caller. Nonetheless, on flops like 6♥7♠8♠, we are grateful to have T♠9♠ and 77 to combat an aggressive button defender, especially when deep stacked. The strengths and weaknesses of each combination, interacting with the defending preflop ranges and each distinct flop, together support a particular bet sizing. Ranges at all stages of the game tree work similarly.

Preflop, a decision to use a uniform bet sizing for an entire opening range from each position is a common choice. It is supported by theory as simple and capturing most potential EV. One can, however, use multiple ranges for preflop holdings. This is commonly seen when someone might add a limping range in addition to a multiple big blind opening size, e.g. on the button in an ante game. Indeed, under certain conditions, splitting preflop ranges across sizings is possible and optimal opening size and range varies with position. Doing this adds both EV and complexity to the strategy and can allow for playing a wider range of hands. Further, any optimal strategy that removes a constraint in the previous strategy will be more profitable. This is a mathematical result in that the new strategy would otherwise reduce to the old strategy if the optimization did not produce a more lucrative plan. Generally, it is helpful to think of our options as an EV landscape and to steer ourselves into the highly positive regions of this surface. For example, there might be game flow reasons to employ one strategy, with lower equilibrium EV, over another because it affords better exploits in a particular poker ecosystem.

This line of thinking led me to assess the value of the hidden information. A bot like PokerSnowie is an opponent that will not adapt its (converged) strategy to an opponent's play. Note, an optimal or GTO approach is literally the strategy that requires no knowledge of the opponent's strategy. Indeed, playing a GTO strategy would mean never adapting to an opponent in any way. When people aspire to such things, they are typically misunderstanding poker. One desires a balanced flexible baseline from which to both defend vs. attackers and simultaneously be poised to exploit weaknesses in the approach of an opponent. Profitable poker is counter strategic. For example, if a live poker game is underway and you see all the players approximating a GTO approach – run away from the table. Practically speaking, developing a robust strategy that shows up with strong hands and viable bluffs for the most common situations that arise is a useful paradigm. The more common the situation is, the more precise the strategy is. Use this strategic flexibility to play against your opponent's strategy. It would be interesting to play vs. PokerSnowie, both knowing its cards and again without that advantage to explicitly quantitate the value of that information. Information is always money in poker.

Next, consider two distinct strategies derived from a simulation, both with similar EV. The kind of strategies with fixed yet distinct bet sizes derived from something like PioSolver. To make things concrete, imagine a common Button vs. Small Blind confrontation where many strategies of similar EV are viable. One can play both strategies, perhaps randomized, and form a new strategy. Imagine picking a random number from 0-1 at the onset of the hand and if it's less than 1/2 we employ strategy A and when the random number is over1/2 then strategy B is used. Distinctly, someone could play strategy A one day and B the next. Both of these present challenges to our opponents in inferring our holdings from our bet size. These considerations imply that playing multiple strategies with weighting coefficients presents an opportunity; strategy A and B can be mixed in any proportion. The logical extension of this is using a distribution of bet sizes for each range in a given game state. Some functional form can be assumed for the distribution of bet sizes. A bet size is then chosen randomly to reproduce the optimized bet sizing probability distribution. A delta function distribution reduces to a single strategy with fixed bet sizes. Further optimizing the distribution itself over function space should represent the complete solution to NLH, in contrast to the approximations of multiple fixed bet sizes.

This is an unexplored avenue. Such a strategy removes an additional constraint and provides for significant additional information hiding. In discussing ranges above, it was implicit that ranges bifurcate and split even into more discrete parts as the game tree is explored. We might have a river spot where our full houses and simulant bluffs bet one sizing, while straights make a significantly smaller wager. Indeed, optimal solvers find such spots to be ubiquitous and even find that one should randomize certain holdings between the bet sizing ranges. In the new paradigm there is a probability associated that the nut-flush in the above spot might sometimes bet with the full house sizing and another percentage of the time with the straights. Current strategies allow for leakage of information that is far from optimal. It is easy to imagine bet sizing distributions with significantly overlapping tails that make our opponent's life in guessing our intentions miserable.

Imagine then that one allowed for bet sizing distributions that are optimized for EV. When choosing to bet, one would randomize from the bet sizing distribution to pick a bet size. A very simple distribution is the existing fixed bifurcated sizing example above that is used commonly. What I am suggesting is a continuous distribution that allows for probability from zero to all-in wagers with associated optimal probabilities. Because this reduces to the extant formalism, any success at optimization suggests this is the actual nature of the solution in NLH, or big bet games more generally. In the example above, a large bet would imply a full house or a bluff from a card combination that mimics full houses by blocking them. In our new paradigm, there may be a small probability the straights and their associated bluffs are betting with this sizing, creating yet more information hiding and additional EV.

Imagine preflop ranges in which each combination was allowed to bet from a distribution of bet sizes that would clearly overlap. Stronger hands could average larger bet sizes but not reveal themselves as weaker holdings would have a fractional probability of betting the same size. The actual bet size would be randomized from the optimized, predetermined bet sizing probability distribution. This offers the potential for increasing the EV of a strategy while maintaining information hiding. Some accomplished players implement intuitive versions of this strategy by using many different bet sizes for a given game state. A formal implementation with randomization would expand from only using these ideas exploitatively. I believe this is the nature of the actual solution to NLH.

Let me reconsider the use of solvers in today's poker climate. Simple solvers like PioSolver are useful in very well-defined situations. They are used primarily in heads-up spots and require the input of assumed ranges and bet sizes. People often, in a single simulation, consider multiple bet sizes simultaneously and chose the one that seems best. That is not a mathematically justified experiment as critical card combinations can be split between ranges. Consider, when facing large bet sizings, the ability to make the nuts in that situation and the frequency associated with having the best possible hand will be essential variables. Bet sizing is also going to interact strongly with range choice, and due to the combinatoric possibilities and the computational demand of the solving algorithms, default choices are ubiquitous. Thus, many bet sizing strategies remain unexplored in a follow-the-leader poker universe. Further, in multi-way situations, disparate stack sizes also are critical to the solution as four identical stacks are very different from a short, medium and two large chip stacks.

Still, in an online environment, all is not lost as the games tend to play in a homogenous environment with 100bb stacks, very similar professional / regular player strategies and most postflop scenarios are engaged heads-up. Thus, a lot of progress has been made in common confrontations like Button vs. Blinds in this well-defined milieu. Still, the results are far from an EV optimized equilibrium even in these idealized situations. The bet sizing space is under sampled, not randomized, and sensitivity to range uncertainties have not been carefully explored. Note, sensitivity analysis for changes in parameter space providing simple guidelines to modify a known strategy are unexplored territory. Poker is played for large amounts of money by smart, talented people. Nonetheless, most of these folks have little training in quantitative methods and there is no incentive and few mechanisms to share formal progress on interesting questions.

Using software like PioSolver for live situations is a different situation altogether. There the uncertainty in ranges is vast if the live game itself is worth playing. Consider, if a recreational player calls an open with JT, whether they include the off-suit combinations changes the mathematics drastically with four suited combinations of JT possible out of sixteen total. Now, the player may or may not have all the or only the suited J9, J8 combinations too -- the situation becomes untenable from a solver derived strategy perspective. I made a large postflop error for 200bb stacks on a 7♥5♠5♥ flop. I correctly assessed that my two opponents would call the $90 preflop raise with a wide variety of suited holdings. However, when the winning player called my all-in flop bet with T5o off-suit, I had drastically underestimated the number of 5s in their preflop range and played my hand poorly as a result. Understanding range construction / interaction in confrontations is far more important than deciding whether to bet 1/2or 2/3 pot in particular situation.

Further consider that many pots become multi-way postflop and one is facing a variety of strategies and stack sizes. Note, a collusive set of independently losing strategies can make the value of your holding change radically. Collusive here is not meant as cheating -- the strategies simply happen to align against your strategy effectively. They can even make it impossible for you to win, even with say the highest EV holding on a flop. The optimal or GTO-like strategy is only guaranteed to win vs. lesser strategies on average when sampled over ranges and possible future game states. A bunch of donkeys can indeed regularly crack your A♥A♠ and these are the games one should endeavor in which to play. Poker worth playing is counter-strategic. If the games get so competitive that you are only playing a set GTO / equilibrium mimicking strategy, find something else to do. The fun in strategic poker is in constructing strategy in new situations on the fly that plays well vs your opponents.

Brian Space is a scientist and professor seeking people to play Quantum Statistical Mechanics for money. He plays poker but is no old man coffee. Remember the GTO strategy ignores that of the other players – you really want to play like that? His poker articles are available on his web site: http://drbrian.space/poker.html

No comments

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